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Gravitational Yukawa Potential from a Yang–Mills Theory for Gravity

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It is shown that the Yang–Mills theory of gravity proposed by us results in a Yukawa-like force as supposed experimentally by E. Fischbach *et al.*

1. INTRODUCTION

Recently we proposed a Yang–Mills gauge theory for gravity on the basis of Minkowski space-time (Dehnen and Ghaboussi, 1985). The reasons for doing this are the difficulties of usual gravity with respect to quantization and unification with the remaining physical interactions. In this connection we have emphasized that in any case quantum or microscopic physics possesses priority and follows directly from very few general first principles, whereas all macroscopic physics, including Einstein's metric theory of gravity, must be deduced from quantum physics in a certain classical limit.

Following the successful line of gauging of compact and unitary internal symmetry groups for describing the electroweak and strong interactions, we have chosen as gauge group for the Yang–Mills theory of microscopic gravity the simplest possibility beyond the $U(1)$ phase-gauge group, namely the $U(2)$ transformation group of the 2-spinors for massless fermions; the mass as a classical concept must be introduced later dynamically by spontaneous symmetry-breaking. Accordingly, in our theory the fermions possess priority and as fundamental fermions one can consider even "preons" or "urs," so that a certain universality of gravity is guaranteed.

Taking additionally into account the usual principle of minimal coupling, we obtained a microscopic Lorentz-invariant Yang–Mills theory, which

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leads in its classical macroscopic limit to Einstein's metric theory of gravity at least in its linearized version: The pseudo-Riemannian metric is built up by the 4-vector gauge potentials with respect to the $SU(2)\times U(1)$ algebra, which we use in its 4×4 representation (Dehnen and Ghaboussi, 1986). In this sense our theory represents in its classical limit a bimetric tetrad formulation of gravitation between (massless) fermionic matter. The interaction of gravity with the other bosonic fields may be included only within a grand unification of all interactions using higher $U(N)$ symmetry groups acting on a higher dimensional spin-isospin space.

Although our theory in its classical limit results in Einstein's theory, on the microscopic level there exist other structures also (Ghaboussi *et al.*, 1987). In this connection it is of interest that recently Fischbach *et al.* (1986) found that a reanalysis of the Eötvös experiment gives rise to an additional Yukawa-like (repulsive) potential besides the usual (attractive) Newtonian potential for the static gravitational interaction. A similar result was found by Holding and Tuck (1984), using geophysical methods. According to this, the effective gravitational potential should have the form

$$V(r)\sim\frac{1}{r}(1+\alpha e^{-r/\lambda}) \tag{1}$$

$[\alpha=-(7.2\pm 3.6)\times 10^{-3},\ \lambda=200\pm 50\ \text{m}]$, from which the force follows

$$|\mathbf{K}|\sim\frac{1}{r^2}+\alpha\left(\frac{1}{r^2}+\frac{1}{r\lambda}\right)e^{-r/\lambda} \tag{2}$$

It is to be supposed that such a Yukawa-like force, which does not exist in Newton's or even Einstein's theory, is a macroscopic relic of microscopic gravity. In this respect the result of Fischbach *et al.* is very important and should be tested as soon as possible.

In this context the question arises of whether—apart from other interpretations of this effect (see, e.g., Gibbons and Whiting, 1981)—our theory of microscopic gravity results in such a Yukawa-like force, and what the predictions of our theory are with respect to experiments. However, because our theory is developed in the first step for massless fermions, one can expect on this level only a qualitative answer.

2. THE MICROSCOPIC THEORY

We repeat so far as is necessary the results of Dehnen and Ghaboussi (1985). The 2-spinor transformation group $U(2)$ is considered as an *internal* symmetry group. Then, using the 4-spinor calculus and consequently the

4×4 representation of the generators τ^a of the group $SU(2)\times U(1)$

$$\tau^a = \frac{1}{2}\begin{pmatrix} \sigma^a & 0 \\ 0 & \sigma^a \end{pmatrix} \tag{3}$$

(σ^0 is the unit matrix and σ^1, σ^2, σ^3 are the Pauli matrices), we obtain the covariant derivative of the spinor ψ:

$$D_\mu\psi = (\partial_\mu + ig\omega_{\mu a}\tau^a)\psi \tag{4}$$

(g is the gauge-coupling constant) with the property

$$[D_\alpha, \gamma^\mu] = 0, \qquad \gamma^{(\mu}\gamma^{\nu)} = \eta^{\mu\nu} \tag{5}$$

(γ^μ are generalized Dirac matrices, $\eta^{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski metric). With respect to the spinor transformation as internal group the spinor ψ is a Lorentz scalar; the adjoint spinor $\bar\psi$ is given by $\bar\psi = \psi^\dagger\zeta$, where the matrix ζ is a Lorentz scalar defined by $(\zeta\gamma^\mu)^\dagger = \zeta\gamma^\mu$ and $[D_\mu, \zeta] = 0$ (ζ and γ^μ are functions of x^ν); a special representation of ζ is $\zeta = \gamma^0$.

The gauge-covariant field strengths are

$$F_{\mu\nu a} = \omega_{\nu a|\mu} - \omega_{\mu a|\nu} - g\varepsilon_a^{\ bc}\omega_{\mu b}\omega_{\nu c} \tag{6}$$

[$\varepsilon_a^{\ bc}$ are structure constants of the group $SU(2)$] satisfying the Bianchi identities:

$$F_{a[\mu\nu|\lambda]} + g\varepsilon_a^{\ bc}F_{b[\mu\nu}\omega_{\lambda]c} = 0 \tag{7}$$

Finally, the field equations following from the minimal coupled gauge-invariant Lagrange density take the explicit form

$$\gamma^\mu(\partial_\mu + ig\omega_{\mu a}\tau^a)\psi = 0 \tag{8}$$

$$\partial_\mu F^{\mu\nu a} + g\varepsilon^{abc}F_b^{\mu\nu}\omega_{\mu c} = 2\pi kg\bar\psi\{\gamma^\nu, \tau^a\}\psi \tag{9}$$

with the "charge" conservation laws:

$$\partial_\nu\left[j^{\nu a} - \frac{g}{4\pi k}\varepsilon^{abc}F_b^{\mu\nu}\omega_{\mu c}\right] = 0 \tag{10}$$

wherein the matter currents are defined by

$$j^{\nu a} = \tfrac{1}{2}g\bar\psi\{\gamma^\nu, \tau^a\}\psi \tag{10a}$$

Here k is a second coupling constant between the two gauge-invariant parts of the Lagrange density and comes out to be $2\hbar G$ (G is the Newtonian gravitational constant), whereas the coupling constant g has the dimension of a reciprocal length and remains undetermined by the classical Einstein limit; it can be determined only experimentally by microscopic effects. The meaning of the quantity $\hbar g$ is that of an elementary gravitational charge, i.e., energy.

The gauge-invariant canonical energy momentum tensor belonging to (8) and (9) is given by

$$T^{\nu}_{\mu} = \tfrac{1}{2} i\hbar[\bar{\psi}\gamma^{\nu} D_{\mu}\psi - (\bar{D}_{\mu}\bar{\psi})\gamma^{\nu}\psi] - \frac{\hbar}{4\pi k}(F_{\mu\alpha a}F^{\nu\alpha a} - \tfrac{1}{4}F^{\alpha\beta}_{a}F^{a}_{\alpha\beta}\delta^{\nu}_{\mu}) \tag{11}$$

with the energy-momentum conservation laws

$$\partial_{\nu} T^{\nu}_{\mu} = 0 \tag{11a}$$

Neglecting surface integrals over the matter field (ψ field) with respect to the conservation laws (10), one finds from (10a) for the change of the 4-momentum of the ψ field with time, using (7) and (9),

$$\partial_0 \int T^{0}_{\mu}(\psi)\, d^3x = \int F_{\mu\alpha a}\hbar j^{\alpha a}\, d^3x \tag{12}$$

where $T^{\nu}_{\mu}(\psi)$ is the gauge-invariant canonical energy-momentum tensor of the matter field given by the bracket on the right-hand side of (11).

Considering an insular and static distribution of matter, we find for the 4-force on the right-hand side of (12)

$$K_{\mu} = F_{\mu 0 a} q^{0a}, \qquad q^{0a} = \hbar \int j^{0a}\, d^3x \tag{13}$$

Of course, in case of massless fermions an exact static distribution of matter is impossible. However, it is to be expected that the structure of the force in (12) and that of the Yang–Mills equation (9) will not be changed for massive fermions; only the current of the matter field will go over from a lightlike to a timelike one. Then for our question the 3-force K_m ($\mu = m = 1, 2, 3$) and consequently the field strengths F_{m0a} according to (13) are significant and must be investigated in detail.

3. THE FIELD STRENGTH

For investigation of the static field strengths $F_{m0(a)}$ (we set the group-set index in a parentheses for clarity) we have to solve the bosonic vacuum field equations following from (9),

$$\partial_m F^{m0(a)} + g\varepsilon^{(a)(b)(c)} F^{n0}_{(b)}\omega_{n(c)} = 0 \tag{14}$$

for the static, spherically symmetric case.[2] In view of the tetrad formulation of the classical Einstein limit, the gauge potentials must satisfy the always possible normalization condition ($|\varepsilon^{(b)}_{(a)}| \ll 1$ in a weak field approximation):

$$\omega_{\mu(a)}\omega^{\mu(b)} = \delta^{(b)}_{(a)} + \varepsilon^{(b)}_{(a)}(x^{\nu}) \tag{15}$$

[2] Latin coordinate indices run from 1 to 3 only.

According to this, the three $SU(2)$ gauge potentials are spacelike and the $U(1)$ gauge potential is timelike upon choosing as group-space metric the Minkowski matrix η_{ab}; this choice is essential for reaching the pseudo-Riemannian structure of the metric in the classical limit. We satisfy the condition (15) for the following by the ansatz:

$$\omega^{\mu}_{(a)} = \delta^{\mu}_{(a)} + A^{\mu}_{(a)} \tag{16}$$

so that $\varepsilon^{(b)}_{(a)}$ and $A^{\mu}_{(a)}$ are connected with one another. Then we get from (14) with respect to (6) for the $U(1)$ part $(a = 0)$

$$A^{0(0)} \sim \frac{1}{r}, \qquad F_{m0(0)} \sim \frac{1}{r^2}\frac{x^m}{r} \tag{17}$$

i.e., the usual Newtonian potential and field strength.

For the $SU(2)$ part $(a = j = 1, 2, 3)$ we obtain from (6) and (16) the field strengths

$$F_{m0(j)} = A_{0(j)|m} - g\varepsilon_{(j)(m)(n)}A_0^{(n)} \tag{18}$$

where we have restricted ourselves to the *linearized* form with regard to $A_{\mu(a)}$. Expression (18) shows that the ansatz (16) is reduced for $A_{\mu(a)} = 0$ to a pure gauge field ($F_{m0(a)} \equiv 0$) at least with respect to the static potentials $A_{0(a)}$, which is necessary for the interpretation of $A_{0(a)}$ as new gauge potentials instead of $\omega_{0(a)}$.

Insertion of (18) into (14) yields the following coupled differential equations for the $SU(2)$ gauge potentials:

$$A^{0(j)|m}_{|m} - 2g\varepsilon^{(j)(m)(n)}A^{0}_{(n)|m} - 2g^2A^{0(j)} = 0 \tag{19}$$

with the asymptotically vanishing solution

$$A^{0(j)} \sim \partial_j[(1/r)\exp(-\sqrt{2}gr)] \tag{20}$$

as the "most spherically symmetric" one. Obviously the $SU(2)$ part leads to Yukawa-like potentials with a range given by the reciprocal gauge coupling constant g^{-1}.

With respect to (13), the Yukawa-like "$SU(2)$ forces" couple to the "changes" $q^{0(j)}$ depending, in view of (10a) and (13), on the spin orientation of matter. Combining (17) and (20), we find with the use of (18) for the 3-force between two elementary point sources with spins in the direction of their connecting line according to (13):

$$K_r = -2\hbar^2g^2G\left[\frac{1}{r^2} + \beta\left(\frac{1}{r^2} + \frac{g}{\sqrt{2}r} + \frac{1}{\sqrt{2}gr^3}\right)\exp(-\sqrt{2}gr)\right] \tag{21}$$

with $\beta = +1$ for parallel and $\beta = -1$ for antiparallel spins. This result has nearly the same structure as the experimentally supposed expression (2), whereby the coupling constant g now can be determined experimentally by the value of λ. Accordingly, the fundamental energy quantity $\hbar g = \hbar\sqrt{2}\lambda$ would be of the order of 10^{-9} eV (cf. also Fischbach *et al.*, 1986). We note that the Yukawa potentials (20) and the Yukawa part of the force (21) do not appear in the static classical Einstein limit of our theory.

4. FINAL REMARK

According to (21), we find a superposition of Coulomb-like and Yukawa-like static forces within our microscopic gauge theory of gravity in qualitative agreement with the result of Fischbach *et al.* Because the effects of the Yukawa potential also depend on the polarization of the test matter, a test of the result of Fischbach *et al.* should be performed with spin-polarized bodies. However, in order to avoid perturbations by the magnetic field of the earth, it may be necessary to use a superconducting shield for the experiments.

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